

# An Evolution Strategy Method for Optimizing Machining Parameters of Milling Operations

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## Abstract

In this paper, an evolutionary strategy (ES) method is introduced as an optimization approach to solve problems in the manufacturing area. The ES method is applied to a case study for milling operations. The results show that it can effectively yield good results.

*Key words:* Evolution Strategy(ES); Milling operation; Design optimization

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Insert Nomenclature here
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## 1 Introduction

In manufacturing industry, it is important to determine the optimal machining parameters in order to maximize total profit rate and to increase the quality of the final product for machining operations. Traditionally, the selection of parameters is carried out by the experience of planners or with the help of machining data handbooks. These may not guarantee the optimum performance and minimization of costs. Therefore, a number of researchers have tried to deal with the optimization of machining parameters using different approaches. Compared with some deterministic methods [1–4] evolutionary algorithms (EAs) are more attractive to engineers since EAs are robust, effective and easy to implement,

Many different types of EAs have been proposed for optimizing machining parameters of milling operations, such as genetic algorithm (GA) [5], simulated annealing (SA) [6], ant colony algorithm [6], immune algorithm [7], and some hybrid algorithms [8, 24–26].

Evolution Strategies(ESs), originally developed by Rechenberg and Schwefel [12, 13], are algorithms which imitate the principle of natural evolution as a method to solve parameter optimization problems [14, 27]. ESs can reject infeasible individuals directly (also called “death penalty”). It is probably the easiest way to handle constraints. Since in the model introduced in sequel, the feasible search space constitutes a reasonably large portion of the whole search space. This strategy seems feasible. Moreover, it is also computationally efficient, because when a certain solution violates a constraint, it is assigned a fitness of zero. Therefore, no further calculations are necessary to estimate the degree of infeasibility of such a solution. To the best knowledge of the authors, no one has used ESs to optimize machining parameters of milling operations. So in this paper, we use an ES to solve this problem. The results show that it can effectively yield good results.

## 2 Mathematical model

Depth of cut, feed rate and cutting speed have the greatest effect on the success of the machining operation. Depth of cut is usually predetermined by the workpiece geometry and operation sequence. It is recommended to machine the workpiece with the required depth in one pass to keep machining

time and cost low, when possible. Therefore the problem of determining the machining parameter is reduced to select the proper cutting speed and feed rate combination. The mathematical model developed by M. Tolouei-Rad et al. [16] is considered in this work. The model is also considered in several papers ([6, 8, 17, 18]).

## 2.1 Objective function

The main focus of this work is to maximize the total profit rate and can be determined by

$$P_r = \frac{S_p - C_u}{T_u}. \quad (1)$$

The unit cost can be represented by

$$\begin{aligned} C_u = & c_{mat} + (c_l + c_o)t_s + \sum_{i=1}^m (c_l + c_o)K_{1i}V_i^{-1}f_i^{-1} \\ & + \sum_{i=1}^m c_{ti}K_{3i}V_i^{(1/n)-1}f_i^{[(w+g)/n]-1} + \sum_{i=1}^m (c_l + c_o). \end{aligned} \quad (2)$$

The unit time to produce a part in the case of multi-tool milling can be defined by

$$T_u = t_s + \sum_{i=1}^m K_{1i}V_i^{-1}f_i^{-1} + \sum_{i=1}^m t_{tci} \quad (3)$$

## 2.2 Constraints

In practice, possible range of cutting speed and feed rate are limited by the following constraints

1. Maximum machine power
2. Surface finish requirement
3. Maximum cutting force permitted by the rigidity of the tool
4. Available feed rate and spindle speed on the machine tool

### 2.2.1 Power

The machining parameters should be selected such that maximum machine power is used. The required machining power should not exceed available motor power. Therefore the power constraint can be written as

$$C_5 V f^{0.8} \leq 1, \quad (4)$$

where

$$C_5 = \frac{0.78K_p W z a_{rad} a}{60\pi d e P_m}. \quad (5)$$

### 2.2.2 Surface finish

The required surface finish  $R_a$ , must not exceed the maximum attainable surface finish  $R_{a(at)}$  under the conditions. Therefore the surface finish for end milling becomes

$$C_6 f \leq 1, \quad (6)$$

where

$$C_6 = \frac{318[\tan(la) + \cot(ca)]^{-1}}{R_{a(at)}}. \quad (7)$$

And for end milling

$$C_7 f^2 \leq 1, \quad (8)$$

where

$$C_7 = \frac{318(4d)^{-1}}{R_{a(at)}}. \quad (9)$$

### 2.2.3 Cutting force

The total cutting force  $F_c$  resulting from the machining operation must not exceed the permitted cutting force  $F_c(\text{per})$  that the tool can withstand. The permitted cutting force for each tool has been considered as its maximum limit for cutting forces. Therefore the cutting force constraints becomes

$$C_8 F_c \leq 1, \quad (10)$$

where

$$C_8 = 1/F_c(\text{per}). \quad (11)$$

### 2.2.4 Speed limits

1. Face milling: 60–120 m/min
2. Corner milling: 40–70m/min
3. Pocket milling: 40–70 m/min
4. Slot milling1: 30–50 m/min
5. Slot milling2: 30–50 m/min

### 2.2.5 Feed rate limits

1. Face milling: 0.05–0.4mm/tooth
2. Corner milling: 0.05–0.5mm/tooth

3. Pocket milling: 0.05–0.5mm/tooth
4. Slot milling1: 0.05–0.5mm/tooth
5. Slot milling2: 0.05–0.5mm/tooth

### 3 The ES method

#### 3.1 Introduction of ESs

Evolution Strategies can be understood as ‘intelligent’ probabilistic search algorithms which are based on the evolutionary process of biological organisms in nature.

The procedure of one type of ES can be described as follows. For the  $\mu$  initial individuals, in optimization terms, each individual in the population is encoded by a real number which represents a possible solution to a given problem. Then a recombination procedure and mutation procedure are executed to produce new ‘offspring’(i.e. *children*) solutions with  $\eta$  ( $\eta > \mu$ ) individuals. The fitness of each individual in ‘offspring’ solutions is evaluated with respect to a given objective function. After evaluation procedure the offspring solutions are sorted and the last  $\eta - \mu$  individuals are deleted. This reproduction-evaluation-selection cycle is repeated until a satisfactory solution is found.

A more comprehensive overview of ESs can be found, e.g. in [14, 19] and references therein. The applications of ES could be found in [21–27]

The basic steps of the procedure can be shown as:

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Generate an initial population;
repeat:
    Recombinant and mutate individuals to produce children;
    Evaluate fitness of the children;
    Select the population from the children;
until a satisfactory solution has been found.
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By means of the above ES procedure, a computing method for optimizing machining parameters of milling operations is described in the following parts of the section.

### 3.2 Representation and fitness function

We denote  $X$  as a vector composed of two arrays representing cutting speed and feed rate, i.e.,  $x_1 = V$  and  $x_2 = f$ ,  $X = (x_1, x_2)$ . The target function (1) is taken as the fitness function in the ES.

### 3.3 Initial population, recombination and mutation

#### 3.3.1 Initial population and Recombination

The initial population consists of  $\mu$  individuals  $\vec{X}(0) = \{X^{(1)}(0), \dots, X^{(\mu)}(0)\}$ , where  $X^{(i)}(0) = (X^i, \sigma^i)$ . The initial components of  $X$  are generated randomly from feasible domains. All initial  $\sigma_i$  are valued by 3.0 here.

Select two individuals:

$$\begin{aligned} (X^1, \sigma^1) &= ((x_1^1, \dots, x_l^1), (\sigma_1^1, \dots, \sigma_l^1)) \\ \text{and } (X^2, \sigma^2) &= ((x_1^2, \dots, x_l^2), (\sigma_1^2, \dots, \sigma_l^2)). \end{aligned}$$

There are two types of recombination operators:

- **discrete**, where the new offspring is

$$(X, \sigma) = ((x_1^{q_1}, \dots, x_l^{q_l}), (\sigma_1^{q'_1}, \dots, \sigma_l^{q'_l}))$$

with  $q_i$  and  $q'_i$  equal either 1 or 2;

- **intermediate**, where the new offspring is

$$(X, \sigma) = ((\alpha x_1^1 + (1-\alpha)x_1^2, \dots, (\alpha x_l^1 + (1-\alpha)x_l^2), (\alpha \sigma_1^1 + (1-\alpha)\sigma_1^2, \dots, \alpha \sigma_l^1 + (1-\alpha)\sigma_l^2)),$$

with  $\alpha \in (0, 1)$ .

By the suggestion of Schwefel [13], the discrete recombination operator is executed on  $X$ , and the intermediate recombination operator is executed on  $\sigma$ .

#### 3.3.2 Mutation

Apply mutation to the offspring  $(X, \sigma)$ , the resulting new offspring  $(X', \sigma')$  is obtained, where

$$\begin{aligned} \sigma'_i &= \sigma_i \cdot \exp(\tau' \cdot N(0, 1) + \tau \cdot N_i(0, 1)) \\ \text{and } x'_i &= x_i + \sigma'_i \cdot N_i(0, 1), \end{aligned} \tag{12}$$

in which,

$N(0, 1)$  — a random number reconciled standard normal distribution;

$N_i(0, 1)$  — a random number reconciled standard normal distribution aimed at  $i$ th component;

$\tau'$  — global coefficient;

$\tau$  — local coefficient;

$l$  — the number of components in  $X$ .

According to the suggestion of Schwefel [13],  $\tau'$  is valued by  $(\sqrt{2l})^{-1}$  and  $\tau$  is valued by  $(\sqrt{2\sqrt{l}})^{-1}$  in the computation.

Notice that  $X$  has box constrains. So the offspring generated by mutation procedures may not be feasible. Then we just let it take the bound value, i.e. if  $x_i > \overline{x_i}$  (or  $< \underline{x_i}$ ), let  $x_i = \overline{x_i}$  (or  $= \underline{x_i}$ ).

The recombination and mutation steps will not stop until  $\eta$  offsprings are generated.

### 3.4 Evaluation, Selection and Stop criterion

We evaluate every  $(X, \sigma)$  by its fitness function and sort them. If an individual is infeasible, the fitness value is 0. Then we choose first  $\mu$  individuals as new parents. In our ES method, we let  $\mu = 15$ ,  $\eta = 105$  and set a variable to record the current best fitness value. If a fitness value is better than the record, we will update the record. The algorithm will stop if the record keeps unchanged after 1000 iterative loops.

## 4 Case Study

The component as shown in Figure 1 is to be produced using a CNC milling machine. It is desirable to find the optimum machining parameters, which result in the maximum profit rate. Specifications of the machine, material and values of constants are given below. Also, the geometric information on the required operations and tools is presented in Tables 1 and 2.

Insert Figure 1 here

### Constants:

$$S_p = \$25$$

$$c_{mat} = \$0.50$$

$c_o = \$1.45$  per min  
 $c_l = \$0.45$  per min  
 $t_s = 2$  min  
 $t_{tc} = 0.5$ min  
 $C = 33.98$  for HSS tools  
 $C = 100.05$  for carbide tools  
 $w = 0.28$   
 $K_p = 2.24$   
 $W = 1.1$   
 $n = 0.15$  for HSS tools  
 $n = 0.3$  for carbide tool  
 $g = 0.14$

**Machine tool data:** Type: Vertical CNC milling machine  $P_m = 8.5$  kW,  $e = 95\%$

**Material data:** Quality: 10L50 leaded steel. Hardness = 225 BHN

Insert Table 1 and Table 2 here.

We use our ES to solve this case and compare it to some other methods in the literature. The results listed in Table 3 show that ES can obtain good results similar to hybrid methods.

Insert Table 3 here.

## 5 Conclusion

In this paper, we used an ES method to optimize machining parameters of milling operations. The results showed that it can effectively give good results and it can be a good alternative in similar problems in engineering.

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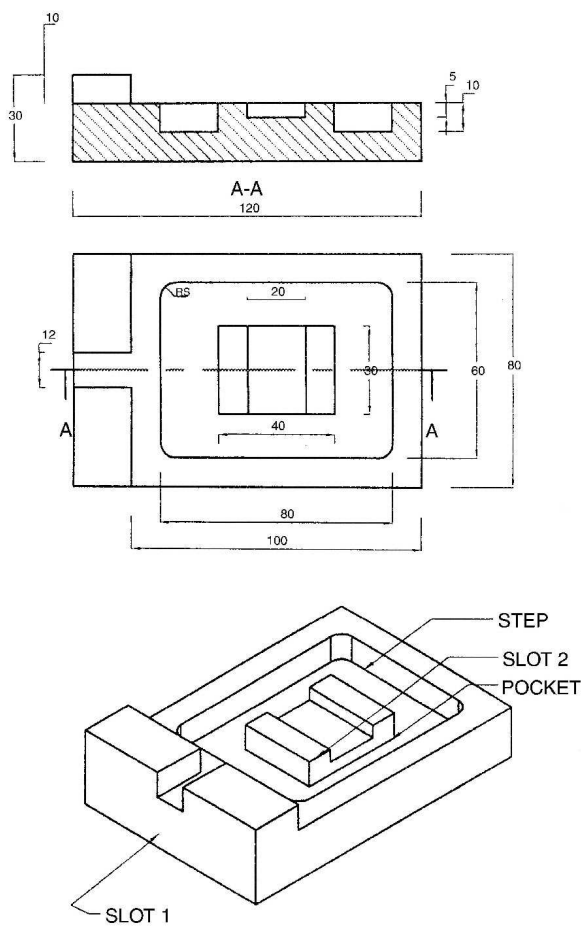
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Fig. 1. An example



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## Nomenclature

$a, a_{rad}$	Axial depth of cut, radial depth of cut (mm)
$ca$	Clearance angle of the tool (degrees)
$C_i$ ( $i = 1, \dots, 8$ )	Coefficients carrying constants values
$c_l, c_o$	Labour cost, overhead cost (\$/min)
$c_m, c_{mat}, c_t$	Machining cost, cost of raw material per part, cost of a cutting tool (\$)
$C_u$	Unit cost (\$)
$d$	Cutter diameter (mm)
$e$	Machine tool efficiency factor
$F$	Feed rate (mm/min)
$f$	Feed rate, (mm/tooth)
$F_c, F_c(\text{per})$	Cutting force, Permitted cutting force (N)
$G, g$	Slenderness ratio, exponent of slenderness ratio.
$K$	Distance to be travelled by the tool to perform the operation (mm)
$K_i$ ( $i = 1, 2, 3$ )	Coefficients carrying constant values
$K_p$	Power constant depending on the workpiece material
$la$	Lead (corner) angle of the tool (degree)
$m$	Number of machining operations required to produce the product
$N$	Spindle speed (rev/min)
$n$	Tool life exponent
$P, P_m$	Required power for the operation, motor power (kW)
$P_r$	Total profit rate (\$/min)
$R$	Sale price of the product excluding material, setup and tool changing costs (\$)
$R_a, R_{a(at)}$	Arithmetic value of surface finish, and attainable surface finish ( $\mu\text{m}$ )
$S_p$	Sale price of the product (\$)
$T, T_u$	Tool life (min), Unit time (min)
$t_m, t_s, t_{tc}$	Machining time, set-up time, tool changing time (min)
$V, V_{hb}, V_{opt}$	Cutting speed, recommended by handbook, optimum (m/min)
$w$	Exponent of chip cross-sectional area
$W$	Tool wear factor
$z$	Number of cutting teeth of the tool

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Table 1

Required machining operation

Operation Number	Operation type	Tool number	$a(\text{mm})$	$K(\text{mm})$	$R_a(\mu\text{m})$
1	Face milling	1	10	450	2
2	Corner milling	2	5	90	6
3	Pocket milling	2	10	450	5
4	Slot milling	3	10	32	—
5	Slot milling	3	5	84	1

Table 2

Tools data

Tool Number	Tool type	Quality	$d(\text{mm})$	$z$	Price(\$)	$la$	$ca$
1	Face mill	Carbide	50	6	49.50	45	5
2	End mill	HSS	10	4	7.55	0	5
3	End mill	HSS	12	4	7.55	0	5

Table 3

Comparison of the results for milling operation

Method	$C_u$ -Unit cost	$T_u$ -Unit time	$P_r$ -Profit Rate
Handbook [20]	\$18.36	9.40 min	0.71/min
Method of feasible direction [16]	\$11.35	5.48 min	2.49/min
Genetic algorithm [17]	\$11.11	5.22 min	2.65/min
Ant colony algorithm [6]	\$10.20	5.43 min	2.72/min
Hybrid particle swarm [18]	\$10.90	5.05 min	2.79/min
Immune algorithm	\$11.08	5.07 min	2.75/min
Hybrid immune algorithm [17]	\$10.91	5.07 min	2.79/min
Hybrid differential evolution algorithm [8]	\$10.90	5.00 min	2.82/min
Evolutionary strategy	\$10.91	5.00 min	2.82/min